# MATHEMATICS FOR POLITICAL SCIENCE

Summer Semester 2022, University of Konstanz

Seminar Time: Monday, 5.00–6.30 PM (17.00–18.30 Uhr)
Seminar Location: C358 (in-person seminar)
Instructor: Jan P. Vogler
Contact: jan.vogler@uni-konstanz.de

Tutorial Time: Wednesday, 3.15–4.45 PM (15.15–16.45 Uhr)
Tutorial Location: G302 (in-person tutorial)
Teaching Assistant: Marius Kaltenbach
Contact: marius.kaltenbach@uni-konstanz.de

## Course Abstract:

The application of rigorous statistical methods is a core aspect of modern political research. Moreover, many key contributions to political science and political economy are based on game theoretic modeling. In order to fully understand these statistical and game theoretic approaches, comprehensive knowledge of the underlying mathematical tools is essential. Therefore, this class introduces students to a number of topics in mathematics that are a prerequisite to advanced classes in methodology: (1) We begin with a quick introduction to the fundamentals of mathematics, including mathematical notation, functions, limits, and other basic topics. (2) Then we study calculus in one dimension, including differentiation, integration, and the identification of extrema. (3) Probability theory is an essential building block of mathematical statistics, which is the reason for us to devote a significant amount of time to this topic. (4) The fourth topic is linear algebra, including systems of equations, Eigenvalues, and Markov chains. (5) Finally, the class closes with an introduction to multivariate calculus. Knowledge of all of these tools will enable the students to subsequently take more advanced methodological classes in statistics and game theory.

## Course Objectives:

By the end of the class, students will be able to:

- Understand the fundamental building blocks of mathematics, including mathematical notation, functions, sequences and series, and more.

- Describe the rules of calculus in one dimension with respect to differentiation, integration, and the evaluation of extrema.
- Elaborate on key components of probability theory, which entail different types of statistical distributions and probability functions, among others.
- Apply the tools of linear algebra to vectors and matrices, solve systems of equations, and find Eigenvalues of matrices.
- Combine several of the mathematical tools we have learned about to identify extrema in multivariate functions.

## Course Requirements:

### Useful Background Knowledge

Basic knowledge of mathematics, up to calculus in one dimension, will be very useful for this class. While students without prior training in calculus are equally encouraged to enroll in the course, the class might be slightly more challenging for them. In general, the course is designed in a way that anyone—even students with little prior training in mathematics or for whom such training is in the distant past—have a chance to succeed. Throughout the semester, I and the teaching assistant will be available to give further advice and guidance to students who want to catch up on any basic subject.

### Preparation for Class, Weekly Readings, and Participation

Students are asked to carefully read and follow the relevant textbook chapters while they are enrolled in the class. While the students are encouraged to study the relevant chapters prior to class to familiarize themselves with terms, notation, and content, they may also choose to first come to class and consult the textbook afterwards, if this approach is more conducive to their learning.

### Written Assignments

There are two types of written assignments in this class. Specifically, students are expected to submit four problem sets and participate in one final examination.

Requirements for the problem sets (50% of the course grade): Four times throughout the semester, the students will receive a problem set and will have one week to complete it. Problem sets will typically include 10 individual tasks that are meant to test students' understanding of the past sessions of the class. They will consist of a mix of easy, medium, and difficult tasks. Students are welcome to discuss possible strategies to solve problems with their peers, but they are required to never copy any solution directly from someone else. Moreover, they are required to write down the submitted solution entirely independently. If we find clear evidence that answers were directly copied between two students, both students will fail the respective assignment. The specific dates for the problem sets can be found in the course schedule below.

Requirements for the final examination (50% of the course grade): The final examination will cover the entire content of the class. It is meant to provide students with an opportunity to demonstrate that they have mastered every subject that they have learned about in the class. The final examination will take 90 minutes. The final examination will take place on **Monday, August 1, 2022.**

### Late Assignment Policy

If you cannot finish an assignment (problem set) on time for a legitimate reason, including, for example, emergencies and illness, please contact me. Please do so ideally before the assignment is due, but at most seven days after the deadline. In case of illness, a statement by your doctor is needed. Depending on the situation, I will provide you with an alternative assignment and/or deadline. If no legitimate reason is provided for late assignments, 10% of the point total for the assignment are subtracted for every day that the assignment is late, and a point total of 0 is awarded if the assignment is more than seven days late.

### Attendance Policy

You are allowed to miss a maximum of three classes. Out of those three, you can miss one class without informing me. However, it is encouraged that you inform me in advance. If you miss more than one class, I expect you to inform me about this in advance. If you miss more than three classes (for any reason), you will automatically fail the class. Please note that this course moves at a very high speed and that missing classes will negatively affect your ability to understand the material of future sessions. Therefore, it is strongly encouraged to avoid missing any classes.

## Grading:

Based on the above requirements, the course grade will consist of the following elements:

- 50%: Four problem sets (12.5% each)
- 50%: Final examination

## Inclusion:

An essential goal of the class is to create an open and welcoming discussion atmosphere. Diversity of opinions, constructive discussion, and mutual respect are at the core of academic discourse and will be key elements of this class. A heterogeneity in backgrounds, experiences, and identities will greatly benefit us by allowing us to learn from each other and expand our thinking. All students are encouraged to voice their opinions and to do so in a way that displays respect for the opinions of other students in the class. Students who believe that these goals are inhibited in any way should contact me so that we can discuss their concerns.

## Academic Integrity:

A second essential goal of mine is to uphold the standards of academic integrity in this class. It is expected that all work submitted is entirely done by the person who submits it. Although it is allowed to discuss strategies while working on problem sets, all final and submitted answers to these problem sets must be written down independently by the individual student who submits the work. If you have any questions about academic integrity, please contact me so that we can discuss them.

## Textbook:

The class is primarily based on the following book. All students are required to acquire a copy of it:

- Moore, W. H., & Siegel, D. A. (2013). *A mathematics course for political and social research*. Princeton University Press. **[Moore & Siegel]**

**Please note that an electronic campus license of the text book has been made available by the library. University of Konstanz students can access this version at the following URL (on campus or via VPN connection):** https://doi.org/10.1515/9781400848614

## Important Dates and Deadlines:

- First class: Monday, April 11, 2022
- Holiday break day I (no class): Monday, April 18, 2022
- Holiday break day II (no class): Monday, June 6, 2022
- Holiday break day III (no class): Monday, June 13, 2022
- First problem set due: Monday, May 9, 2022
- Second problem set due: Monday, May 30, 2022
- Third problem set due: Monday, July 4, 2022
- Fourth problem set due: Monday, July 18, 2022
- Last class: Monday, July 18, 2022
- Final examination: Monday, August 1, 2022 (Room C230)
  (11.15 AM to 12.45 PM; 11.15–12.45 Uhr)

## Tutorial:

Once per week, there will be a 90-minute tutorial led by the teaching assistant. Participation in the tutorial is a regular aspect of the class and thus expected, except for cases of illness or emergency. The tutorial will be organized in the following way: For 30 minutes, the teaching assistant will review key concepts from that week's class. If we were unable to cover any material from a relevant chapter in class, the teaching assistant may also cover (parts of) it in the tutorial. For another 30 minutes, the teaching assistant will go through prepared exercises with the students. For the last 30 minutes, students can ask questions (e.g., "Can you explain this concept again?", "How do I solve this problem?"). If there is anything specific that you would like the teaching assistant to cover in the tutorial, please email the teaching assistant at least 6 hours before the beginning of the tutorial so that he/she/they can prepare accordingly.

## Office Hours:

If you would like to speak with me, please contact me by email to set up an appointment. In the email, please include the specific reason why you would like to speak with me and provide me with at least three different dates at and time frames during which you are available.

***Course Schedule Begins on the Following Page.***

# COURSE SCHEDULE:

## PART I: INTRODUCTION TO THE CLASS AND TO MATHEMATICS FOR POLITICAL SCIENCE

The first part of the class introduces the students to the class schedule and to the fundamentals of mathematics. In the first session, we discuss why we need to study mathematics if we want to do cutting-edge political research. Moreover, this part of the class covers (1) variables and constants, (2) sets, (3) operators, (4) relations, (5) levels of measurement, (6) notation, (7) an introduction to proofs, (8) basic properties of arithmetic, (9) algebra review, (10) computational aids, (11) functions, (12) preference relations and utility functions, (13) sequences and series, (14) limits, (15) sets, (16) continuous functions.

1. **Introduction and Course Overview: "Mathematics for Political Science"** **(April 11, 2022)**

   **The Relevance of Mathematics for Modern Research in Political Science** (Online Class)

Required Reading:

- No specific required readings for the first class, but I will circulate some research articles that use statistical research methods to underscore the importance of mathematics to modern political research. Please pick one that is of greatest interest to you and read it more carefully.

## — NO CLASS ON APRIL 18, 2022 (HOLIDAY BREAK)! —

2. **Building Blocks, Part I** **(April 25, 2022)**

   **Preliminaries + Algebra Review**

Required Readings:

- **Moore & Siegel,** chap. 1 ("Preliminaries")
- **Moore & Siegel,** chap. 2 ("Algebra Review")

3. **Building Blocks, Part II** **(May 2, 2022)**

   **Functions, Relations, & Utility + Limits and Continuity, Sequences & Series, More On Sets**

Required Readings:

- **Moore & Siegel,** chap. 3 ("Functions, Relations, and Utility")

- **Moore & Siegel,** chap. 4 ("Limits and Continuity, Sequences and Series, and More On Sets")

## PART II: CALCULUS IN ONE DIMENSION

The second part of the class deals with calculus in one dimension. Specifically, this part covers (1) introduction to calculus, (2) the derivative, (3) rules for differentiation, (4) derivatives of functions, (5) the definite integral, (6) indefinite integrals and the fundamental theorem of calculus, (7) computing integrals, (8) rules of integration, (9) extrema, (10) higher-order derivatives, concavity, and convexity, and (11) finding extrema.

**4. Calculus, Part I: Fundamentals (Differentiation)** **(May 9, 2022)**

**Introduction to Calculus and the Derivative + The Rules of Differentiation**

Required Readings:

- **Moore & Siegel,** chap. 5 ("Introduction to Calculus and the Derivative")
- **Moore & Siegel,** chap. 6 ("The Rules of Differentiation")

***Problem Set 1 Due at the Beginning of Class on May 9, 2022!***

**5. Calculus, Part II: The Integral** **(May 16, 2022)**

Required Reading:

- **Moore & Siegel,** chap. 7 ("The Integral")

**6. Calculus, Part III: Extrema in One Dimension** **(May 23, 2022)**

**(Class led by Marius Kaltenbach)**

Required Reading:

- **Moore & Siegel,** chap. 8 ("Extrema in One Dimension")

## PART III: PROBABILITY THEORY

The third part of the class deals with probability theory. Specifically, it covers (1) basic probability theory, (2) computing probabilities, (3) specific measures of probabilities, (4) the distribution of a single variable, (5) sample distributions, (6) empirical joint and marginal distributions, (7) the probability mass function, (8) the cumulative distribution function, (9) probability distributions and statistical modeling, (10) expectations of random variables, (11) continuous random variables, (12) expectations of continuous random variables, (13) important continuous distributions for statistical modeling.

**7. Probability, Part I: An Introduction to Probability** **(May 30, 2022)**

Required Reading:

- **Moore & Siegel,** chap. 9 ("An Introduction to Probability")

***Problem Set 2 Due at the Beginning of Class on May 30, 2022!***

## — NO CLASS ON JUNE 6, 2022 (HOLIDAY BREAK)! —

## (THE TUTORIAL WILL TAKE PLACE IN THIS WEEK!)

## — NO CLASS ON JUNE 13, 2022 (HOLIDAY BREAK)! —

**8. Probability, Part II: Discrete Distributions** **(June 20, 2022)**

Required Reading:

- **Moore & Siegel,** chap. 10 ("Discrete Distributions")

**9. Probability, Part III: Continuous Distributions** **(June 27, 2021)**

Required Reading:

- **Moore & Siegel,** chap. 11 ("Continuous Distributions")

# PART IV: LINEAR ALGEBRA

The fourth part of the class deals with linear algebra. Specifically, it covers (1) scalars and vectors, (2) matrices, (3) properties of vectors and matrices, (4) matrix illustration of OLS estimation, (5) vector spaces, (6) solving systems of equations, (7) Eigenvalues, Eigenvectors, and matrix decomposition, and (8) Markov chains and stochastic processes.

**10. Introduction to Linear Algebra & Vector Spaces and Systems of Equations** **(July 4, 2022)**

Required Reading:

- **Moore & Siegel,** chap. 12 ("Fun with Vectors and Matrices")
- **Moore & Siegel,** chap. 13 ("Vector Spaces and Systems of Equations") (esp. pp. 304–309)

***Problem Set 3 Due at the Beginning of Class on July 4, 2022!***

**11. Eigenvalues and Markov Chains** **(July 11, 2022)**

Required Reading:

- **Moore & Siegel,** chap. 13 ("Vector Spaces and Systems of Equations") (esp. pp. 310–324)
- **Moore & Siegel,** chap. 14 ("Eigenvalues and Markov Chains")

# PART V: MULTIVARIATE CALCULUS

The fifth and final part of the class deals with multivariate calculus. Specifically, it covers (1) functions of several variables, (2) calculus in several dimensions, (3) concavity and convexity redux, (4) unconstrained optimization, (5) constrained optimization with equality constraints, (6) constrained optimization with inequality constraints, (7) properties of the maximum and minimum, and (8) implicit differentiation.

**12. Introduction to Multivariate Calculus** **(July 18, 2022)**

Required Readings:

- **Moore & Siegel,** chap. 15 ("Multivariate Calculus")
- **Moore & Siegel,** chap. 16 ("Multivariate Optimization")

***Problem Set 4 Due at the Beginning of Class on July 18, 2022!***

## Dates and Deadlines at the End of the Semester:

- Final Examination: Monday, August 1, 2022 (Room C230)
  (11.15 AM to 12.45 PM; 11.15–12.45 Uhr)